Table 1 Predicted and measured conductances for example problem

	Conductances (W/m 2K)			
	Predicted			Measured
$10^3 \frac{P}{H}$	h_c	h_g	h_j	h_j
0.165	170	1800	1970	2300
0.265	260	1880	2140	2430
0.364	360	1940	2300	2560
0.498	480	2010	2490	2800
0.651	620	2080	2700	3030
0.809	760	2140	2900	3240
1.130	1050	2230	3280	3690
1.459	1330	2310	3640	4070
1.788	1620	2380	4000	4570
2.091	1880	2440	4320	4910
2.577	2290	2520	4810	5500
3.162	2780	2600	5380	6220

for $2 \leq Y/\sigma \leq 4$ and $1 \leq M/\sigma < \infty$. The maximum error associated with the simple correlations of Eqs. (16) and (17) is about 2% with this respect to the numerically integrated results.

Example Problem

The agreement between the simple correlations summarized here and real experimental data can be illustrated by considering an example test case from the recent work of Hegazy.[7] In particular, the test results for a pair of 304 Stainless Steel specimens contacting in a nitrogen environment at 570 Torr are summarized in Table 1. The predicted conductances obtained from the simple correlations of this work are also summarized in Table 1 based upon the thermophysical data supplied by Hegazy.[7] Note that $M/\sigma \approx 0.093$ in this case for all the results reported in Table 1. It is evident, at least for this example, that the agreement between the predicted and experimental conductances is excellent.

Conclusions

New correlations for the gap conductance integral in the conforming rough surface model have been presented. Accurate estimations of the thermal joint conductance of many new practical contact problems can now be made quickly by combining these new correlations with the simple contact conductance correlations also provided in this Note.

Acknowledgments

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Steady Conjugate Heat Transfer in Fully Developed Laminar Pipe Flows

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Introduction

FORCED convective heat transfer in a laminar pipe flow has long been recognized as a basic heat-transfer problem and has been extensively studied in the past. Recently, the effects of the conduction heat transfer in the pipe wall on the convection heat transfer in the flow have received considerable attention.[1-6] Close examination of these recent investigations reveals that either the radial thermal resistance of the wall is neglected or the flow is assumed to be uniform without any rigorous justification.

In the present study, we consider the conjugate heat transfer in laminar flow through a long circular pipe that is directly heated over a finite length. Both axial and radial heat conduction in the pipe wall and fluid are accounted for in the analysis for the entire pipe (directly heated and unheated). Emphasis is placed on the comparison between the results of the present study and those neglecting the radial wall resistance.[2] An exponential finite-difference scheme is employed to solve the coupled energy equations.

Analysis

The system to be studied corresponds to a fluid flowing in an infinitely long circular pipe ($-\infty < x < \infty$). A uniform heat flux q''_{wo} is applied at the outer surface of the pipe over a finite length ($0 \leq x \leq \ell$). The upstream ($-\infty < x < 0$) and downstream ($\ell < x < \infty$) regions of the heating zone are thermally well insulated, and the flow enters the pipe with uniform velocity u_e and uniform temperature T_e from the far upstream region ($x \to -\infty$). In this study we are interested in the thermal interactions between the conduction heat transfer in the pipe wall and the convection heat transfer in the fluid through the fluid wall interface.

To simplify the analysis, the thermophysical properties of the fluid are assumed to be temperature-independent, and the flow is hydrodynamically fully developed in the region where significant heat transfer is present. Accordingly, the steady conjugate heat transfer in the system considered can be described by the following basic equations in dimensionless form:

Energy equation for the fluid:

$$Pe(1-\eta^2)\frac{\partial\theta_f}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta_f}{\partial\eta}\right) + \frac{\partial^2\theta_f}{\partial\xi^2} \tag{1}$$

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Energy equation for the wall:

$$\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta_w}{\partial\eta}\right)+\frac{\partial^2\theta_w}{\partial\xi^2}=0 \tag{2}$$

The associated boundary and interfacial conditions are

$$\eta=0,\qquad \partial\theta_f/\partial\eta=0,\qquad -\infty<\xi<\infty \tag{3a}$$

$$\eta=1,\qquad \partial\theta_f/\partial\eta=K\partial\theta_w/\partial\eta \tag{3b}$$

and

$$\theta_f=\theta_w,\qquad -\infty<\xi<\infty$$

$$n=\beta,\qquad \partial\theta_w/\partial n=1/K,\qquad 0\le\xi\le L$$

$$0,\qquad \text{otherwise} \tag{3c}$$

$$\xi\to-\infty\qquad \theta_f=\theta_w=0 \tag{3d}$$

$$\xi\to+\infty\qquad \partial\theta_f/\partial\xi=\partial\theta_w/\partial\xi=0 \tag{3e}$$

In writing the preceding equations, the following nondimensional variables were introduced:

$$\xi=x/R_i,\qquad \eta=r/R_i$$

$$\theta=(T-T_e)/(q''_{wo}\,R_i/k_f)$$

$$Pe=2R_iu_e/\alpha_f,\qquad K=k_w/k_f$$

$$\beta=R_o/R_i,\qquad L=\ell/R_i \tag{4}$$

Here x and r are the axial and radial coordinates, R_i and R_o are the inside and outside radii, k is the thermal conductivity, and α is the thermal diffusivity. Subscripts f and w, respectively, denote the properties for the fluid and wall.

Four governing nondimensional groups appear for the problem, namely, the Peclet number Pe, the ratio of inside and outside radii β, the wall-to-fluid conductivity ratio K, and the dimensionless heated length L.

The dimensionless interfacial heat flux is evaluted by

$$Q_{wi}=\frac{R_i}{R_o}\frac{q''_{wi}}{q''_{wo}}=\frac{K}{\beta}\frac{\partial\theta_w}{\partial\eta}\bigg|_{\eta=1} \tag{5}$$

Solution Method

Since the governing differential equations are of conjugate nature, numerical procedures were employed to find the solution. To procure enhanced accuracy, a fully implicit, exponential scheme[7,8] was adopted to treat the axial energy trnasport in the flow in which the conduction and convection are simultaneously present. The heat conduction equation for the pipe wall and the radial conduction in Eq. (1) were discretized by the conventional central differences. Nonuniform grids were used to account for the drastic variations of θ_f and θ_w in some regions. In the radial direction, the solid region ($1\le\eta\le\beta$) was equally divided into 24 intervals, while in the fluid region ($0\le\eta\le 1$), 21 nodes were employed with the step size adjacent to the interface being smallest and successively enlarged by 15%. Axially, 121 nodal points are distributed in the domain of computation ($-L/2\le\xi\le 3L/2$). The upstream, directly heated, and downstream regions are, respectively, divided into 30, 60, and 30 intervals. The step sizes near the beginning ($\xi=0$) and end ($\xi=L$) are smallest with $\Delta\xi=0.152$, and they are successively made larger by 10% in both positive and negative ξ directions. It is noted in the numerical computation that the extension of the computational domain to $\xi=-L/2$ in the upstream and $\xi=3L/2$ in the downstream is sufficient to meet the conditions specified in Eqs. (3d) and (3e).

The coupled, finite-difference equations for θ_f and θ_w were cast into a single tridiagonal matrix form along each constant ξ line with the interfacial conditions, Eq. (3b), incorporated in these equations. The matrix can be efficiently inverted by the Thomas algorithm.[8] The solutions for θ_f and θ_w were marched from $\xi=-L/2$ to $\xi=3L/2$, and the procedure was repeated until the summation of the relative errors in temperature over all nodes between two consecutive iterations was below 0.001.

In the following section the verification of the preceding numerical scheme is described. The predicted bulk temperature for the asymptotic case of infinitely high Pe and K, large L, and β extremely close to unity is in excellent agreement with the analytic result $-\theta_b=4\xi/Pe$. Also, the errors in the overall energy balance for all cases studied are less than 0.1%. Finally, the predicted asymptotic temperature in the far downstream region of the directly heated section deviates with the exact result by about 0.7%.

Results and Discussion

In this study, numerical computations covering wide ranges of governing parameters ($Pe=2$–100, $\beta=1.01$–1.8, $K=0.2$–30, $L=10$–200) have been performed. Results are particularly presented herein to illustrate the importance of the radial heat conduction in the pipe wall. To be more specific, a comparison between the predictions from the present study and those without considering the radial wall conduction by Faghri and Sparrow[2] is made. The results for the distributions of the interfacial heat flux and the temperature at interface are displayed in Figs. 1 and 2 for various K. For Q_{wi} just outside the directly heated region, the discrepancy between the two predictions becomes larger for small K, but the differences are not too significant for all cases studied. In the direct heating region ($0<\xi<L$), the two predictions give essentially the same interfacial heat fluxes except those for $K=0.2$. Unlike Q_{wi}, substantial difference is noted for the variation of θ_{wi} for every K. In short, the error in predicting Q_{wi} caused by neglecting the radial temperature gradient in the pipe wall is not too serious, but the resulting error in predicting θ_{wi} is important. This result can be understood by realizing that the interfacial heat flux, as the heat-transfer coefficient, is a quantity related to the gross thermal characteristics of the system, and hence can be well predicted by the simplified methods. A typical well-known example is the results obtained for the Nusselt number distributions for a

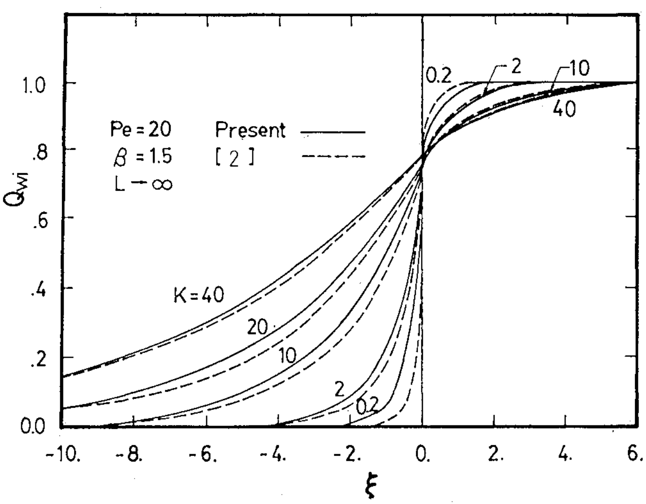

Fig. 1 Comparison of the interfacial heat fluxes in the neighborhood of the beginning end of the heated section.

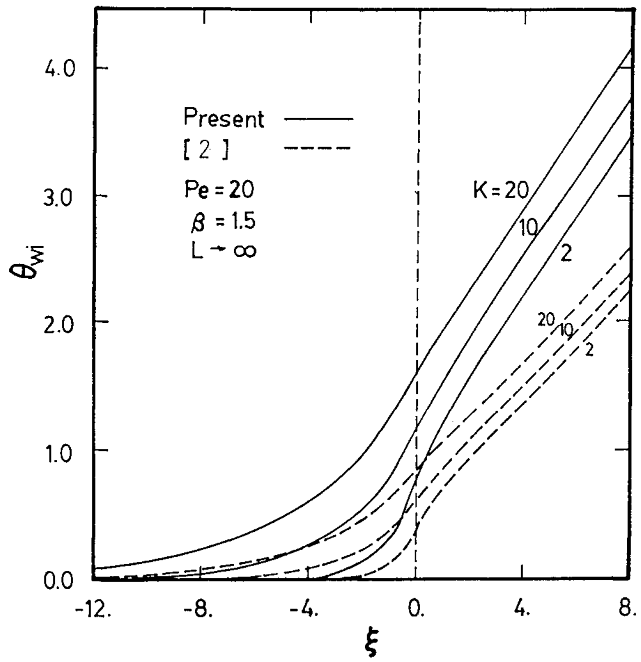

Fig. 2 Comparison of the interfacial temperatures in the neighborhood of the beginning end of the heated section.

laminar forced convection over a flat plate by the integral method.[9]

According to the present investigation, it can be concluded that radial wall conduction becomes important for a system with a high wall-to-fluid thermal conductivity ratio K, thick pipe wall (β much greater than unity), high Peclet number Pe, and short heating length L.

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Evaporation from a Drop Suspended in an Electric Field

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Introduction

SINGLE droplet evaporation has been a subject of intensive investigation for years, but the effects of an electric field upon the evaporation rates have received very little attention. In recent years there has been a renewed interest in the applications of an electric field to promote the heat and mass transfer processes. This favorable enhancement is due to the induced motion inside and outside the drop caused by the charge built up in the neighborbood of the drop surface, which leads to the formation of electrical stresses. The electrically generated motion was first observed and calculated by Taylor[1] and later was employed in the studies of heat and mass transfer by various investigators. Morrison[2] solved the conjugate heat and mass transfer problem at high Peclet numbers, in which he found that the transfer coefficient is proportional to the field strength. At low Peclet numbers, the dominant region of heat and mass transport is not thin enough for boundary layer approximations to be valid; hence, the analysis of the transfer processes has to be started with the convective diffusion equation. Griffith and Morrison[3] expanded the dependent quantities in powers of Peclet number, and then followed the usual perturbation procedure to obtain the temperature distribution for systems with transfer resistance in the continuous phase. In the high Peclet number range, Sharper and Morrison[4] presented numerical solutions to external problems using the finite-difference numerical method.

As a droplet is moving slowly under the influences of gravity and an electric field, a translational motion is superimposed on the Taylor flowfield, and this results in much more complicated flow patterns. Chang et al.[5] and Chang and Berg[6] derived the hydrodynamical solutions for drops translating at low and intermediate Reynolds numbers, respectively. They then incorporated this information to study the heat and mass transfer for both the inside and the outside of the fluid sphere, but their results are strictly applicable at high Peclet numbers. Chung et al.[7] examined the convective diffusion equation for an internal heat transfer problem by an alternating direction implicit method.

The enhancement of heat and mass transfer due to an electric field is a good indication that the rate of evaporation also would be increased because the mass of liquid lost by evaporation is a linear function of the transfer coefficient. The purpose of this Note, therefore, is to extend the work of Morrison[2] to investigate the evaporation from a spherical fluid droplet suspended in an electric field.

Mathematical Model

In this section we shall formulate a model for a pure liquid drop with initial radius R_0 and temperature T_0 suspended in a hot quiescent environment at temperature T_∞, and with the two-phase system subjected to a uniform electric field of strength E. With the physical system so described, we are now

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